

## Suggested cases to add to Exact Analytical Conduction Toolbox

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### A. HEAT EQUATION

1. 1D Cartesian: X30B0T1, X30B0Gx5T0
2. Two-dimensional Cartesian (steady or unsteady)
  - Slab: X00Y33 heated at boundary, internally, initially
  - Semi-slab X30Y33, heated at boundary, internally, initially
  - Quarter space X30Y30, heated at boundary, internally, initially
  - Rectangle X33Y33, heated at boundary, internally, initially
3. Three dimensional Cartesian (steady or unsteady)
  - Infinite plate (or slab) X00 Y00 Z33
  - Half space X30 Y00 Z00
  - Quadrant X30 Y30 Z00; Octant X30 Y30 Z30
4. 1D Cylindrical cases
  - a. solid: R03B1T0, R03B0T1, R03B0T5, R03B0Gr5T0
  - b. hollow: R33B01T0, R33B00Gr5T0
5. 2D Cylindrical cases (steady or unsteady)
  - a. finite cylinder with circular symmetry RIJZML
  - b. infinite slab with circular symmetry R00ZML
  - c. full circle RIJphi00; wedge of circle RIJphiKL
6. 3D Cylindrical cases
  - a. full cylinder RIJz33phi00, R0Jz33phi00
  - b. wedge of cylinder RIJz33phiKL, R0Jz33phiKL
7. Spherical
  - a. 1D solid: RS02B0T5, RS02B0Gr5T0, RS03B1G1T1
  - b. 1D hollow shell: RS33B00Gr5T0, RS33B00T5
  - c. 2D or 3D applications?

## B. BIOHEAT EQUATION (fin equation)

### 1. Cartesian cases

- a. 1D: slab X33B11F1Gx5T1  
semi-infinite XJOB1F1Gx5T1  
infinite X00F1Gx5T5
- b. 2D: rectangle XIJB00YKLB00F1G1T1  
slab XIJB00Y00F1G1T1
- c. 3D: parallelepiped XIJB00YKLB00ZMNB00F1G1T1  
slab XIJB00Y00Z00F1G1T1, etc.

### 2. Cylindrical cases

- a. 1D: solid R03B0F1G1T1  
hollow RIJB00F1G1T1, RIJB10F1T0  
infinite body R00Gr5F1T5
- b. 2D: halfspace with circular symmetry, XJ0R00F1, variously heated  
semi-infinite solid cylinder XJOB0R03B0F1G1T1  
semi-infinite hollow cylinder XJOB0R33B0F1G1T1  
disk R03B0phi00F1G1T1, variously heated
- c. 3D cylinder R03B0Z33B00Phi00F1G1T1, variously heated

### 3. Spherical cases

- a. 1D: solid RS03B0F1Gr5Tr5  
hollow RSIJB00F1Gr5T5
- b. 2D or 3D applications?

### C. HEAT EQUATION WITH CONVECTIVE TERM

#### 1. Cartesian, steady

a. 2D:  $\frac{u}{\alpha} T_x = T_{yy}$  (boundary layer)

$$\frac{u}{\alpha} T_x = T_{xx} + T_{yy} \quad (\text{X00Y33V1, flow between plates})$$

b. 3D:  $\frac{u}{\alpha} T_x = T_{xx} + T_{yy} + T_{zz}$  (X00Y33Z33V1, rectangular channel)

#### 2. Cartesian, transient

a. 1D:  $\frac{1}{\alpha} T_t + \frac{u}{\alpha} T_x = T_{xx}$  (X00V1G5T5)

b. 2D:  $\frac{1}{\alpha} T_t + \frac{u}{\alpha} T_x = T_{yy}$  (boundary layer)

$$\frac{1}{\alpha} T_t + \frac{u}{\alpha} T_x = T_{xx} + T_{yy} \quad (\text{X00Y33V1T0, flow between plates})$$

#### 3. Cylinder, steady

a. 2D:  $\frac{u}{\alpha} \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right)$  (boundary layer)

$$\frac{u}{\alpha} \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + T_{xx} \quad (\text{X00R03V1 tube; X00R33V1 annulus})$$

b. 3D:  $\frac{u}{\alpha} \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + T_{xx} + \frac{1}{r^2} T_{\theta\theta}$  (tube with angular dependence)

#### 4. Cylinder, transient. Etc.